Meanings as proposals: an inquisitive approach to exhaustivity

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Structure

1. Problems for existing accounts
2. Exhaustivity and disjunction
3. Exhaustivity and quantification

Part I: Problems for existing accounts

Some examples

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Previous work

- Alonso-Ovalle, L. (2008).
- Chierchia, G., Fox, D., & Spector, B. (2008).
- Groenendijk, J., & Roelofsen, F. (2009).
- Horn, L. (1972).
- Rooij, R. van, & Schulz, K. (2006).
- Sauerland, U. (2005).
- Spector, B. (2007).
- ...

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Dialogue is a cooperative enterprise.

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Part II: Exhaustivity and disjunction

Semantics

Definition: Inquisitive semantics (Ciardelli, *et al.*, 2009)

- $[p] = \{\{w \in \mathbf{W} | w(p) = 1\}\}$
- $[\bot] = \{\emptyset\}$
- $[\varphi \vee \psi] = [\varphi] \cup [\psi]$
- $[\varphi \wedge \psi] = \{\alpha \cap \beta | \alpha \in [\varphi], \beta \in [\psi]\}$
- $[\varphi \to \psi] = \ldots$

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Definition: Entailment

$A \models B \iff \text{for some } C, B \sqcap C = A$

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$A \propto B \iff \text{for some } C, B \cup C = A \iff B \subseteq A$

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Fact: Attention and entailment

For an initiative φ and response ψ s.t. $\varphi \propto \psi$:
ψ unattends a possibility iff $\psi \not\models \varphi$.

Some conversational maxims

Maxim of Quality

Maxim of Relation

Maxim of Attention (new)

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Only attend a set of possibilities if you consider them individually possible, and their union necessary.

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Maxim of Quality

Only attend a set of possibilities if you consider them individually possible, and their union necessary.

Maxim of Relation

Only attend relevant possibilities.

Maxim of Attention (new)

Do not attend/unattend a possibility without reason.

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(5) My father or mother will be home $\not\models$ only one of them

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Definition: Exhaustivity implicature

For an initiative φ and response ψ, s.t. $\varphi \propto \psi$:
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Examples:

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Definition: Exhaustivity suggestion

$\varphi \mathrel{\sim} \bigcup\{A \mid \text{for some } \psi, \varphi \propto \psi, \textbf{size}([\psi]) = 1, \psi \mathrel{|\!\!\sim}_{\varphi} A\}$

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- $p \vee q \mid\!\sim [\neg q \vee \neg p] \qquad (1)$

Implicatures and suggestions

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For an initiative φ and response ψ, s.t. $\varphi \propto \psi$:
$\psi \mid\!\approx_{\varphi} \bigcap\{\overline{\alpha} \mid \alpha \in [\varphi], \alpha \cap \bigcup[\psi] \notin [\psi] \text{ or } \alpha = \emptyset\}$

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$\varphi \mid\!\sim \bigcup\{A \mid \text{for some } \psi, \varphi \propto \psi, \textbf{size}([\psi]) = 1, \psi \mid\!\approx_{\varphi} A\}$

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- $p \mid\!\approx_{p \vee q \vee r} [\neg q \wedge \neg r]$
- $p \vee q \mid\!\sim [\neg q \vee \neg p]$ (1)
- $p \vee q \vee r \mid\!\sim [(\neg q \wedge \neg r) \vee (\neg p \wedge \neg r) \vee (\neg p \wedge \neg q)]$ (2)

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Examples:

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- $p \mathrel{|\!\!\mid\!\!\sim_{p \vee q \vee r}} [\neg q \wedge \neg r]$
- $p \vee q \mathrel{|\!\!\sim} [\neg q \vee \neg p]$ (1)
- $p \vee q \vee r \mathrel{|\!\!\sim} [(\neg q \wedge \neg r) \vee (\neg p \wedge \neg r) \vee (\neg p \wedge \neg q)]$ (2)
- $p \vee q \vee (p \wedge q) \mathrel{|\!\!\sim} [\neg q \vee \neg p \vee \top]$ (4)

Part III: Exhaustivity and quantification

Semantics

Definition: F.O. Inquisitive Semantics (Ciardelli, 2010)

- $[P(t_1, \ldots, t_n)]_g = \{\{w \in \mathbf{W} | \langle [t_1]_{w,g}, \ldots, [t_n]_{w,g} \rangle \in [P]_w\}\}$
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Work in progress: numerals

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(6) A man came to me $\mathrel{|\!\sim}$ only one man came

(7) n men came to me $\mathrel{|\!\sim}$ only n men came

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(6) A man came to me $\mid\!\sim$ only one man came

(7) n men came to me $\mid\!\sim$ only n men came

(8) At least n men came to me $\not\mid\!\sim$ only n men came

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- $n :: \lambda P \lambda Q.\exists X(P(\check{}X) \wedge Q(\check{}X) \wedge |X| = n)$

Semantics

Definition: F.O. Inquisitive Semantics

- $[P(t_1, \ldots, t_n)]_g = \{\{w \in \mathbf{W} | \langle [t_1]_{w,g}, \ldots, [t_n]_{w,g} \rangle \in [P]_w\}\}$
- $[\bot]_g = \{\emptyset\}$
- $[\varphi \vee \psi]_g = [\varphi]_g \cup [\psi]_g$
- $[\varphi \wedge \psi]_g = [\varphi]_g \sqcap [\psi]_g$
- $[\varphi \rightarrow \psi]_g = \ldots$
- $[\exists X.\varphi]_g = \bigcup_{D' \subseteq D} [\varphi]_{g[X/D]}$
- $[\forall X.\varphi]_g = \sqcap_{D' \subseteq D} [\varphi]_{g[X/D]}$

- $a :: \lambda P \lambda Q.\exists X(P(\check{}X) \wedge Q(\check{}X) \wedge |X| = 1)$
- $n :: \lambda P \lambda Q.\exists X(P(\check{}X) \wedge Q(\check{}X) \wedge |X| = n)$
- at least $n :: \lambda P \lambda Q.\exists X(P(\check{}X) \wedge Q(\check{}X) \wedge |X| \geq n)$

Work in progress: vague numerals?

(9) Some men came to me $\mid\!\sim$ many men did not come

Work in progress: vague numerals?

(9) Some men came to me $\mid\!\sim$ many men did not come

(10) Many men came to me $\mid\!\sim$ some men did not come

Work in progress: vague numerals?

(9) Some men came to me $\mid\!\sim$ many men did not come

(10) Many men came to me $\mid\!\sim$ some men did not come

(11) Most men came to me $\mid\!\sim$ a minority did not come

Work in progress: vague numerals?

(9) Some men came to me $\mid\!\sim$ many men did not come

(10) Many men came to me $\mid\!\sim$ some men did not come

(11) Most men came to me $\mid\!\sim$ a minority did not come

$$\text{Some} :: \lambda P \lambda Q.\exists X(P(\check{}X) \wedge Q(\check{}X) \wedge |X| \approx \text{prototype}(\text{Some } P\ Q))$$

Conclusion

A uniform account of exhaustivity in terms of:

- Compliant responses
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Future work: conditionals, modals, content words.

Fin.

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